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ON POSITIONAL SIMULATION IN DYNAMIC SYSTEMS*

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The problem of simulating delays and controls in a dynamic system described by ordinary differential-difference equations is examined. The simulation is carried out in real time by the feedback principle on the basis of information and the current phase states of the system, measurable with a certain error. The simulation algorithm proposed — an algorithm for reconstructing the unknown delays and controls — is a regularizing one in the sense that the simulation results become better the less the measurement errors in the system's phase positions. The ideological source of the proposed method of solving the problem is Krasovskii's extremal aiming principle /1, 2/. The paper extends the investigation in /3, 4/ and touches on /1—5/.

1. We have the following controlled system:

$$x^{\cdot}(t) = f(t, x(t), x(t-\tau(t)), u(t)), \quad t_0 \leqslant t \leqslant \vartheta \tag{1.1}$$

$$x(t_0+s) = x_0(s), \quad s \in [-\beta, 0] \tag{1.2}$$

Here x is the phase vector, u is the control, τ is the delay, and t is time. We have a measuring device with a certain error, which estimates the system's phase state $x(t)$ at the current instants $t \in [t_0, \vartheta]$. The result of the estimation is the vector $\psi(t)$. It is required to find a positional algorithm /1/ which on the basis of this information up to the instant ϑ reconstructs (in a certain sense) the delay and the control, as well as the initial state from which the system's motion started at the instant t_0. This is the meaningful description of the problem.

Let us refine the formulation of the problem. Suppose we are given the compacta $P \subset R^m$, $Q \subset R^n$ (R^ν is a ν-dimensional space with Euclidean norm denoted by $\|\cdot\|$), the n-dimensional function $f(t, x, y, u)$, $t \in [t_0, \vartheta]$, $x, y \in R^n$, $u \in R^m$, and the numbers $\alpha \geqslant 0$, $\beta > \alpha$. Let

$$\|\psi(t) - x(t)\| \leqslant \beta_*, \quad t_0 \leqslant t \leqslant \vartheta \tag{1.3}$$

where $x(t)$ is the motion of system (1.1), (1.2). The (Lebesgue-) measurable functions $\tau(t)$: $[t_0, \vartheta] \to [\alpha, \beta]$ and $u(t)$: $[t_0, \vartheta] \to P$ and the Borel function $x_0(s)$: $[-\beta, 0] \to Q$ are unknown. We are given a convex, positive-homogeneous, continuous function $\omega(w)$: $R_+ \to R_+$ ($R_+ = \{w \in R^1 \mid w \geqslant 0\}$), $\omega(0) = 0$, continuously differentiable in the domain $\{w > 0\}$.

From a prescribed $\varepsilon > 0$ we are required to find a positional procedure for forming the control $u_\varepsilon[t] = u(t, \psi_t(s))$, the delay $\tau_\varepsilon[t] = \tau(t, \psi_t(s))$ ($\psi_t(s) = \psi(t+s)$, $s \in [-\beta, 0]$) and the initial state $z_{0\varepsilon}[s]$, $-\beta \leqslant s \leqslant 0$, such that

$$\omega(\|x(t) - z[t]\|) \leqslant \varepsilon, \quad t_0 \leqslant t \leqslant \vartheta \tag{1.4}$$

where $z[t]$ is a solution of the differential equation

$$z^{\cdot}[t] = f(t, z[t], z[t-\tau_\varepsilon[t]], u_\varepsilon[t]), \quad t_0 \leqslant t \leqslant \vartheta$$

$$z[t_0+s] = z_{0\varepsilon}[s], \quad -\beta \leqslant s \leqslant 0$$

To realize the proposed algorithm it is sufficient to know, at instants $t \in [t_0, t_0+\beta)$, only the vector $\psi(t)$. We note as well that the function $z_{0\varepsilon}[s]$ will now be defined up to the instant $t_0+\beta$.

In Sect.1 we indicate a method of solving the problem, suitable for computer realization. In Sects.2 and 3 we discuss the convergence of $\tau_\varepsilon[t]$ and $z_{0\varepsilon}[s]$ to the actual $\tau(t)$ and $x_0(s)$ as $\varepsilon \to 0+$. In Sect.4 we give two typical examples.

The problems of reconstructing the delays, the controls and the initial functions from the a priori known set $\{C\psi(t_1), C\psi(t_2), \ldots, C\psi(t_N)\}$ ($t_i \in [t_0, \vartheta]$ are fixed instants of time) were considered in /6—8/, where only linear systems were examined in /6, 7/, which in /8/ non-linear systems were investigated using the method of linearization and sensitivity theory. An important feature of the problems considered in the present paper is that here we are concerned with the position reconstruction of these quantities when there is no information on the function $\psi(t)$, $t_0 \leqslant t \leqslant \vartheta$ available in advance.

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We will denote by $\{\mu\}$ and $\{\mu_1\}$ the sets of all Borel probability measures $\mu_t(dv)$, weakly measurable in t, concentrated for all $t \in [t_0, \vartheta]$ in the sets $[\alpha, \beta]$ and Q, respectively /9/; $S(a) \subset R^n$ is a closed unit sphere of radius a with centre at zero; $\|\cdot\|_C$ is the norm in the space $C^n[t_0, \vartheta]$ of n-dimensional functions continuous on $[t_0, \vartheta]$; X is the bundle of all solutions of system (1.1), (1.2), corresponding to all possible Lebesgue-measurable functions $u(t)$: $[t_0, \vartheta] \to P$; $\tau(t)$: $[t_0, \vartheta] \to [\alpha, \beta]$ and Borel functions $x_0(s)$: $[-\beta, 0] \to Q$; $X_{t_*} \subset R^n$ is the section of the bundle X by the hyperplane $t = t_*$; Δ is a partitioning of the interval $[t_0, \vartheta]$ by the points $t_0 < t_1 < \ldots < t_{l(\Delta)} = \vartheta$ $(l(\Delta) < \infty)$, with diameter $\delta(\Delta) = \max_i (t_{i+1} - t_i)$; $V(x)$ is the gradient of the function $\omega_*(x) = \omega(\|x\|)$; a prime denotes transposition.

In what follows we assume that the function $f(t, x, y, u)$ is continuous in t, u, locally Lipschitz in x, y and uniformly in $t \in [t_0, \vartheta]$, $u \in P$ satisfies the growth condition

$$\|f(t, x, y, u)\| \leqslant c_1 + c_2\|x\| + c_3\|y\| \tag{1.5}$$

where c_1, c_2, c_3 are certain constants. We have

Lemma 1.1. Set X is compact /10/ in $C^n[t_0, \vartheta]$.

We will write an algorithm for solving the problem. We set

$$\begin{aligned} &z_{0\varepsilon}[0] = x_0 \\ &z_{0\varepsilon}[s] = x_{\varepsilon i},\ s \in [t_i - t_0 - \beta,\ t_{i+1} - t_0 - \beta),\quad t_{i+1} < t_0 + \beta \\ &u_\varepsilon[t] = u_{\varepsilon i},\ \tau_\varepsilon[t] = \tau_{\varepsilon i},\ t \in [t_i,\ t_{i+1}) \end{aligned} \tag{1.6}$$

where the vectors $x_0, x_{\varepsilon i}, u_{\varepsilon i}$ and the numbers $\tau_{\varepsilon i}$ for $t = t_i$ are chosen from the following conditions. For $t = t_0$ $\tau_{\varepsilon 0} = \beta$, $u_{\varepsilon 0}$ is an arbitrary vector from P, x_0 and $x_{\varepsilon 0}$ are any of the vectors satisfying the equalities

$$\begin{aligned} &\omega(\|\psi(t_0) - x_0\|) = \min_{x \in Q} \omega(\|\psi(t_0) - x_0\|) \\ &V'(\psi(t_0) - x_0) f(t_0, \psi(t_0), x_{\varepsilon 0}, u_{\varepsilon 0}) = \\ &\quad \max_{x \in Q} V'(\psi(t_0) - x_0) f(t_0, \psi(t_0), x, u_{\varepsilon 0}) \end{aligned} \tag{1.7}$$

if $\psi(t_0) \notin Q$; if, however, $\psi(t_0) \in Q$, then $x_{\varepsilon 0}$ is any vector from Q. At the instant $t = t_i \in (t_0, t_0 + \beta)$ the vectors $\tau_{\varepsilon i}$, $u_{\varepsilon i}$, $x_{\varepsilon i}$ satisfy the relations

$$\begin{aligned} &\tau_{\varepsilon i} = \beta,\quad V'(\psi(t_i) - r(t_i)) f(t_i, \psi(t_i) \\ &x_{\varepsilon i}, u_{\varepsilon i}) = \max_{\substack{u \in P \\ x \in Q}} V'(\psi(t_i) - r(t_i)) f(t_i, \psi(t_i), x, u) \end{aligned} \tag{1.8}$$

if $\psi(t_i) \neq r(t_i)$; otherwise, $u_{\varepsilon i}$ and $x_{\varepsilon i}$ are any vectors from P and Q, respectively. Here $r(t)$, $t_0 \leqslant t \leqslant t_i$, is a solution of the equation

$$\begin{aligned} &r^\cdot(t) = f(t, \psi(t_j), x_{\varepsilon j}, u_{\varepsilon j}) \\ &t_{j-1} \leqslant t < t_j,\quad j = 1, \ldots, i,\quad r(t_0) = x_0 \end{aligned} \tag{1.9}$$

If $t_i \geqslant t_0 + \beta$, then $u_{\varepsilon i}, \tau_{\varepsilon i}$ are chosen, when $\psi(t_i) \neq g(t_i)$, from the condition

$$V'(\psi(t_i) - g(t_i)) f(t_i, \psi(t_i), \psi(t_i - \tau_{\varepsilon i}), u_{\varepsilon i}) = \max_{\substack{\xi \in [\alpha, \beta] \\ u \in P}} V'(\psi(t_i) - g(t_i)) f(t_i, \psi(t_i), \psi(t_i - \xi), u) \tag{1.10}$$

while when $\psi(t_i) = g(t_i)$, $u_{\varepsilon i}$, $\tau_{\varepsilon i}$ are any vectors from P and $[\alpha, \beta]$. Here $g(t)$, $t_0 + \beta \leqslant t \leqslant t_i$, is a solution of the equation

$$\begin{aligned} &g^\cdot(t) = f(t, \psi(t), \psi(t - \tau_{\varepsilon j}), u_{\varepsilon j}) \\ &t_0 + \beta \leqslant t_j \leqslant t < t_{j+1} \leqslant t_i,\quad g(t_0 + \beta) = r(t_0 + \beta) \end{aligned} \tag{1.11}$$

The procedure indicated is carried out up to the instant $t_i = \vartheta$.

Theorem 1.1. For any $\varepsilon > 0$ we can find $\gamma_1 > 0$ and $\gamma_2 > 0$ such that for any number $\beta_* \leqslant \gamma_1$ and partitioning Δ with diameter $\delta(\Delta) \leqslant \gamma_2$ inequality (1.4) holds if $z_{0\varepsilon}[s]$, $u_\varepsilon[t]$ and $\tau_\varepsilon[t]$ have been defined as in (1.6).

Proof. We will adopt the following notation:

$$\begin{aligned} &a = \sup\{\|x\| \mid x \in X_t,\ t \in [t_0, \vartheta],\ x \in Q\} \\ &b_1 = \sup\{\|f(t, x, y, u)\| \mid t \in [t_0, \vartheta],\ x \in X_t,\ y \in Q \cup \{\textstyle\bigcup_{t-\beta \leqslant \xi \leqslant t} X_\xi\},\ u \in P\} \\ &b(\delta) = \sup\{\|V(x)\| \mid x \in S(2a - \delta)\},\quad \delta < 2a \\ &\varphi(\alpha) = \sup\{\|f(\xi, x, y, u) - f(t, x, y, u)\| \mid u \in P,\ x, y \in S(a) \\ &t, \xi \in [t_0, \vartheta],\ |t - \xi| \leqslant \alpha\} \end{aligned}$$

$y(t) = r(t)$ for $t \in [t_0, t_0+\beta]$, $y(t) = g(t)$ for $t \in (t_0+\beta, \vartheta]$, $r(t)$ and $g(t)$ are solutions of (1.9) and (1.11), respectively. We will fix $\varepsilon_1 > 0$ and show that numbers $\delta_1 > 0$ and $\beta_1 > 0$ exist such that for all $\beta_* \leqslant \beta_1$ and $\delta(\Delta) \leqslant \delta_1$

$$\omega(\| x(t) - y(t) \|) \leqslant \varepsilon_1, \quad t_0 \leqslant t \leqslant \vartheta \tag{1.12}$$

In view of the properties of ω we find $\delta_2 > 0$ such that

$$\omega(w) \leqslant \varepsilon_2 = {}^1/_2\varepsilon_1 \tag{1.13}$$

for $w \in [0, \delta_2]$. Taking into account the estimate

$$\| y(t) - x(t) - y(t_i) + \psi(t_i) \| \leqslant \beta_* + 2b_1\delta(\Delta), \quad t \in [t_i, t_{i+1}] \tag{1.14}$$

we select the numbers $\delta_3 \in (0, \delta_2)$, $\beta_3 > 0$, starting from the inequality

$$\beta_3 + 2\delta_3 b_1 < {}^1/_2\delta_2 \tag{1.15}$$

Let us assume that inequality (1.12) is violated for some $t \in [t_0, \vartheta]$. By virtue of the continuity of the functions $\omega(w)$ and $\| x(t) - y(t) \|$ we can construct an interval $[t_*, t^*] \subset [t_0, \vartheta]$ in which

$$\omega(\| x(t) - y(t) \|) > \varepsilon_2 \tag{1.16}$$

Let $i_1 = \max\{i \in [0: l(\Delta)] \mid t_i \leqslant t_*\}$, $i_2 = \min\{i \in [0: l(\Delta)] \mid t_i \geqslant t^*\}$. Then when $\delta(\Delta) \leqslant \delta_3$ and $\beta_* \leqslant \beta_3$ the estimates

$$\| y(t) - x(t) \| \geqslant {}^1/_2\delta_2, \quad t_{i_1} \leqslant t \leqslant t^*; \quad \| y(t_i) - \psi(t_i) \| \geqslant {}^1/_2\delta_2, \quad i_1 \leqslant i < i_2 \tag{1.17}$$

$$\omega(\| y(t_i) - x(t_i) \|) \leqslant \max\{\varepsilon_2, \ \omega(\| x(t_0) - x_0 \|)\} \leqslant \varepsilon_2 \tag{1.18}$$

follow from (1.13)–(1.16). In the interval $[t_{i_1}, t^*]$ the function $\omega(\| x(t) - y(t) \|)$ is absolutely continuous: therefore, for $t \in [t_i, t_{i+1}] \subset [t_{i_1}, t_{i_2}] \cap [t_0, t_0+\beta]$

$$\omega(\| y(t) - x(t) \|) \leqslant \omega(\| y(t_i) - x(t_i) \|) + \int_{t_i}^{t} V'(x(\xi) - y(\xi))\{f(\xi, x(\xi), x(\xi - \tau(\xi)), u(\xi)) - f(\xi, \psi(t_i), x_{\varepsilon i}, u_{\varepsilon i})\}\, d\xi \tag{1.19}$$

Since the function $V(x)$ is continuous in the domain $\{x \in R^n \mid \| x \| > 0\}$ and Lemma 1.1 holds, from $\varepsilon_3 > 0$ we can find $\delta_4 = \delta_4(\varepsilon_3) > 0$ such that for $\delta(\Delta) \leqslant \delta_4$, $\xi \in [t_i, t_{i+1}]$, $i_1 \leqslant i < i_2$

$$\| V(x(t_i) - y(t_i)) - V(x(\xi) - y(\xi)) \| \leqslant \varepsilon_3 \tag{1.20}$$

Moreover, this estimate is uniform with respect to all $x(\cdot), y(\cdot)$ satisfying (1.17) and all partitionings Δ with diameter $\delta(\Delta) \leqslant \delta_4$. In addition, uniformly in $y \in Q$, $x(\cdot) \in X$, $u \in P$

$$\| f(\xi, x(\xi), y, u) - f(t, x(t), y, u) \| \leqslant L \mid \xi - t \mid + \varphi(\mid \xi - t \mid) \tag{1.21}$$

where L is the Lipschitz constant of the function $f(t, x, y, u)$ in a domain $D \subset R^n$ in which all the motions being examined remain. Thus, for $\beta_* \leqslant \beta_3$ and $\delta(\Delta) \leqslant \delta_5 = \min\{\delta_3, \delta_4\}$, from (1.19)–(1.21) we obtain

$$\omega(\| y(t) - x(t) \|) \leqslant \omega(\| y(t_i) - x(t_i) \|) + \int_{t_i}^{t} V'(\psi(t_i) - y(t_i))\{f(t_i, \psi(t_i), x(\xi - \tau(\xi)), u(\xi)) - f(t_i, \psi(t_i), x_{\varepsilon i}, u_{\varepsilon i})\}\, d\xi + \overline{\varphi}(\varepsilon_3, \beta_*, \delta(\Delta))\delta(\Delta) \tag{1.22}$$

$$\overline{\varphi} = 4\varepsilon_3 b_1 + b({}^1/_2\delta_2)\{2\varphi(\delta(\Delta)) + L\delta(\Delta) + L\beta_*\}$$

In turn, from this and from (1.18), (1.19) it follows that

$$\omega(\| y(t) - x(t) \|) \leqslant \omega(\| y(t_i) - x(t_i) \|) + \overline{\varphi}(\varepsilon_3, \beta_*, \delta(\Delta))\delta(\Delta), \quad t \in [t_i, t_{i+1}] \tag{1.23}$$

Let $t \in [t_i, t_{i+1}] \subset [t_{i_1}, t_{i_2}] \cap [t_0+\beta, \vartheta]$. Then, as for (1.22), we derive an estimate from which, by virtue of (1.10), (1.11), we obtain

$$\omega(\| y(t) - x(t) \|) \leqslant \omega(\| y(t_{i_1}) - x(t_{i_1}) \|) + \{\overline{\varphi}(\varepsilon_3, \beta_3, \delta_5) + b({}^1/_2\delta_2)L\beta_*\}(\vartheta - t_0), \quad t \in [t_{i_1}, t_{i_2}] \tag{1.24}$$

We will assume that the numbers ε_3, β_3 and δ_5 satisfy the inequality

$$\{\overline{\varphi}(\varepsilon_3, \beta_3, \delta_5) + b({}^1/_2\delta_2)L\beta_*\}(\vartheta - t_0) \leqslant \varepsilon_2 \tag{1.25}$$

Then from (1.18), (1.24) we obtain a contradiction with (1.12). Hence, if as β_1 and δ_1 we take $\beta_3 = \beta_3(\varepsilon_1)$ and $\delta_5 = \delta_5(\varepsilon_1)$ defined in accord with the procedure indicated above, then inequality (1.12) is satisfied.

Let us now estimate the change in the function $\omega(\| z[t] - x(t) \|)$ as t varies within the limits t_0 to ϑ . Since $\omega(w)$ is an increasing function, we have

$$\omega(\|z[t]-y(t)\|) \leqslant \omega\left(kL\int_{t_0}^{t}\|z[\xi]-x(\xi)\|\,d\xi + aL(t-t_0)\,\delta(\Delta) + 2\beta_* L(t-t_0);\ k = \begin{cases}1, & t\in[t_0, t_0+\beta]\\ 2, & t\in(t_0+\beta, \vartheta]\end{cases}\right. \tag{1.26}$$

From (1.12) and (1.26), taking into account the convexity and positive homogeneity of the function $\omega(w)$, on the strength of Jensen's integral inequality /10/ we have

$$\omega(\|z[t]-x(t)\|) \leqslant 2L\int_{t_0}^{t}\omega(\|z[\xi]-x(\xi)\|)\,d\xi + \varepsilon_1 + \omega(a(\vartheta-t_0)\,\delta(\Delta) + 2\beta_*(\vartheta-t_0))$$

Consequently

$$\omega(\|z[t]-x(t)\|) \leqslant \{\varepsilon_1 + (\vartheta - t_0)\,\omega(a\delta(\Delta) + 2\beta_*)\}\times\exp 2L(t-t_0)$$

Now we can set $\gamma_1 = \min\{\delta_5(\varepsilon_1), \delta_6(\varepsilon)\}$, $\gamma_2 = \min\{\beta_3(\varepsilon_1), \beta_4(\varepsilon)\}$, assuming that ε_1, β_4 and δ_6 satisfy the inequalities

$$\varepsilon_1 \exp 2L(\vartheta - t_0) \leqslant {}^1/_2\varepsilon$$
$$\omega(a(\vartheta-t_0)\,\delta_6 + 2\beta_4(\vartheta-t_0))\exp 2L(\vartheta-t_0) \leqslant {}^1/_2\varepsilon$$

The theorem is proved.

Suppose a certain sequence of numbers $\{\varepsilon_j\}$, $\varepsilon_j \to 0+$ as $j\to\infty$, is specified. From Theorem 1.1 we have

Corollary 1.1. Number sequences $\{\delta_j\}$ and $\{\beta_j^*\}$, $\delta_i \to 0+$, $\beta_j^* \to 0+$ as $j\to\infty$, exist such that for any partititionings $\Delta_j = \{t_i^{(j)}\}$, $i = 0, \ldots, l(\Delta_j) < \infty$ with diameters $\delta(\Delta_j) \leqslant \delta_j$ and for any functions $\psi_j(t)$, $\|\psi_j(t) - x(t)\|_C \leqslant \beta_j^*$, the inequalities $\|z_j[t] - x(t)\|_C \leqslant \varepsilon_j$ hold.

Here $z_j[t]$ is a solution of the equation $(\tau[t;\varepsilon_j] = \tau_{\varepsilon_j}[t])$

$$z^\cdot[t] = f(t, z[t], z[t-\tau[t;\varepsilon_j]], u_{\varepsilon_j}[t]) \tag{1.27}$$
$$t_0 \leqslant t \leqslant \vartheta,\quad z[t_0+s] = z_{0\varepsilon_j}[s]$$

while the functions $z_{0\varepsilon_j}[s], u_{\varepsilon_j}[t], \tau_{\varepsilon_j}[t]$ are determined from $\psi_j(t)$ by the algorithm presented above.

2. Suppose that the functions $x_0(s)$ and $u(t)$ are known and that the signal $\psi(t)$ enters without noise ($(\beta_* = 0)$ in (1.3)). The question arises as to the convergence of the sequence $\tau[t;\varepsilon_j]$ to $\tau(t)$. Statements yielding a satisfactory answer to this question in certain cases are proved below. Subsequently, for brevity we will omit the symbol $u(t)$ in (1.1).

Theorem 2.1. Let a unique measure $\mu_t^*(dv) \in \{\mu\}$ exist such that

$$x(t) = x(t_0) + \int_{t_0}^{t}\int_{[\alpha,\beta]} f(\xi, x(\xi), x(\xi - v))\,\mu_\xi^*(dv)\,d\xi,\quad t_0\leqslant t\leqslant\vartheta \tag{2.1}$$

Then $\tau[t;\varepsilon_j] \to \tau(t)$ in $L_2^n[t_0, \vartheta]$.

Proof. Assume that we can find a subsequence $\{\tau[t;\varepsilon_{j(i)}]\}$ $(j(i)\to\infty$ as $i\to\infty)$ such that $\tau[t;\varepsilon_{j(i)}] \nrightarrow \tau(t)$ in $L_2^n[t_0,\vartheta]$. Let $\{\mu_t^{(j)}(dv)\} \in \{\mu\}$ be a sequence with the property

$$\int_{t_0}^{t}\int_{[\alpha,\beta]} f(\xi, z_j[\xi], z_j[\xi - v])\,\mu_\xi^{(j)}(dv)\,d\xi = \int_{t_0}^{t} f(\xi, z_j[\xi], z_j[\xi - \tau[\xi;\varepsilon_j]])\,d\xi,\quad t_0\leqslant t\leqslant\vartheta \tag{2.2}$$

(here and below $z_j[t]$ is a solution of system (1.27)). Then, without loss of generality we can assume that

$$\mu_t^{(j(i))}(dv) \to \bar\mu_t(dv)\quad \text{weakly}\quad [9],\ \bar\mu_t(dv) \neq \mu_t^*(dv) \tag{2.3}$$

Because, if $\bar\mu_t(dv) = \mu_t^*(dv)$, then, as follows from the results of /11/, $\tau[t;\varepsilon_{j(i)}] \to \tau(t)$ in $L_2^n[t_0, \vartheta]$. Therefore, in $C^n[t_0,\vartheta]$

$$\lim_{i\to\infty}\left\{x(t_0) + \int_{t_0}^{t}\int_{[\alpha,\beta]} f(\xi, x(\xi), x(\xi-v))\,\mu_\xi^{(j(i))}(dv)\,d\xi\right\} = \tag{2.4}$$
$$\lim_{i\to\infty} z_{j(i)}[t] = x(t_0) + \int_{t_0}^{t}\int_{[\alpha,\beta]} f(\xi, x(\xi), x(\xi-v))\,\bar\mu_\xi(dv)\,d\xi$$

However, by hypothesis, a unique measure satisfying (2.1) exists. Consequently, by virtue of (2.3), (2.4) we can find $t_* \in [t_0, \vartheta]$ such that $\lim_{i\to\infty} z_{j(i)}[t_*] \neq \psi(t_*)$. This contradicts the fact that $z_j[t] \to x(t)$ in $C^n[t_0, \vartheta]$ (see Corollary 1.1). The theorem is proved.

We will denote by L the set of measures $\mu_t(dv) \in \{\mu\}$ satisfying (2.1).

Theorem 2.2. Let any measure $\mu_t(dv) \in L$ be concentrated. Then the sequence $\{\tau[t; \varepsilon_j]\}$ converges in $L_2^n[t_0, \vartheta]$ to L.

The proof is the same as the proof of Theorem 2.1.

Theorem 2.3. Let $f(t, x, y) = A(t)y + f(t, x)$ and let the matrix $A(t)$, $t_0 \leqslant t \leqslant \vartheta$ be non-singular. Then $x(t - \tau[t; \varepsilon_j]) \to x(t - \tau(t))$ weakly in $L_2^n[t_0, \vartheta]$.

3. Let us find the conditions for the convergence of $z_{0\varepsilon_j}[s]$ to $x_0(s)$ when the delay $\tau(t) = \beta = \text{const}$ is known and $\beta_* = 0$. Here we assume $\vartheta = t_0 + \beta$.

Theorem 3.1. Let a unique measure $\mu_t(dv) \in \{\mu_1\}$ exist such that

$$x(t) = x(t_0) + \int_{t_0}^{t}\int_{Q} f(\xi, x(\xi), \vartheta)\,\mu_\xi(dv)\,d\xi, \quad t_0 \leqslant t \leqslant \vartheta \tag{3.1}$$

Then $z_{0\varepsilon_j}[s] \to x_0(s)$ in $L_2^n[-\beta, 0]$.

Theorem 3.2. Let the conditions of Theorem 2.3 be satisfied. Then $z_{0\varepsilon_j}[s] \to x_0(s)$ weakly in $L_2^n[-\beta, 0]$.

The proofs of Theorems 3.1 and 3.2 are analogous to those of theorems 2.1 and 2.3, respectively.

Notes. *1°.* The assertions in this paper remain valid if a compactum $T_* \subset [\alpha, \beta]$ is known such that $\tau(t) \in T_*$ for all $t \in [t_0, \vartheta]$. In this case the set T_*, and not $[\alpha, \beta]$, will figure in all our constructions.

2°. When $T_* = \{\alpha, \beta\}$ condition (2.1) will be satisfied if for any $t \in [t_0, \vartheta]$, $x(t + s_1) \neq x(t + s_2)$ for all $s_1, s_2 \in [\alpha, \beta]$ and for any $y_1, y_2 \in S(a)$, $y_1 \neq y_2$

$$f(t, x(t), y_1) \neq f(t, x(t), y_2) \quad \text{for almost all} \quad t \in [t_0, \vartheta] \tag{3.2}$$

3°. Condition (3.1) will be satisfied if set Q consists of two points and (3.2) is valid.

4. The problem of reconstructing the initial function $x_0(s)$ was modelled for the system

$$x^{\cdot}(t) = 2{,}5\cos x(t) + 1{,}7\sin x(t-1) \tag{4.1}$$

$$0 \leqslant t \leqslant 1, \quad x \in R^1, \quad Q = \{0, 2\}$$

It was assumed that $\delta(\Delta) = t_{i+1} - t_i = 0.005$. The trajectory $x(t)$ of system (4.1), corresponding to $x_0(s) = 2$ for $s \in [-1/2, 0]$ and to $x_0(s) = 0$ for $s \in [-1, -1/2)$, was calculated by Euler's method.

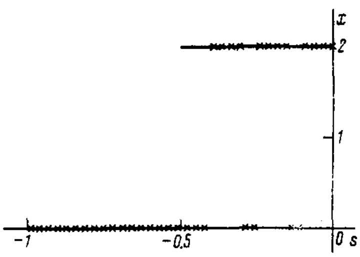

Fig.1

Fig.1 shows the functions $z_{0\varepsilon_1}[s]$ (the solid lines) and $z_{0\varepsilon_2}[s]$ (the crosses) corresponding to $\psi(t_i) = x(t_i) + 0.01$ and $\psi(t_i) = x(t_i) - 0.1$. The estimate $|x(t) - z_1[t]|_C \leqslant 0.02$ $(|x(t) - z_2[t]|_C \leqslant 0.1)$. is valid for the motions $x(t)$ and $z_1[t]$ $(z_2[t])$, where $z_1[t]$ $(z_2[t])$ corresponds to the initial function $z_{0\varepsilon_1}[s]$ $(z_{0\varepsilon_2}[s])$. The problem of reconstructing $x_0(s)$, $u(t)$ and $\tau(t)$. was simulated for the system

$$x_1^{\cdot}(t) = x_2(t), \quad x_2^{\cdot}(t) = -36x_1(t) - u(t)\,x_1(t - \tau(t)) - 9x_2(t) \tag{4.2}$$

$$0 \leqslant t \leqslant 4, \quad \alpha = 0, \quad \beta = 2, \quad P = \{u \mid |u| \leqslant 3\}$$

$$Q = \{(x_1, x_2) \mid |x_1| \leqslant 5, \quad |x_2| \leqslant 1\}$$

It was assumed that $\delta(\Delta) = t_{i+1} - t_i = 0.01$, while the trajectory $x(t)$ of system (4.2) was generated by the control $u(t) = -3\sin t$, the delay $\tau(t) = 1 + \sin 0{,}2t$ and the initial function $x_{01}(s) = 4{,}1 + 0{,}9\cos s$, $x_{02}(s) = \sin s$.

Figs.2 and 3 show the functions $x_{01}(s), x_{02}(s), \tau(t), u(t)$ (the solid lines) and the functions $z_{0\varepsilon}[s] = \{z_1[s], z_2[s]\}, u_\varepsilon[t], \tau_\varepsilon[t]$ ($z_1[s]$ and $u_\varepsilon[t]$ are denoted by dashes, while $z_2[s]$ and $\tau_\varepsilon[t]$ are denoted by crosses). The estimate

$$\|x(t) - z[t]\| \leqslant 0,\ 1, \quad 0 \leqslant t \leqslant 4$$

is valid for the motions $x(t)$ and $z[t]$

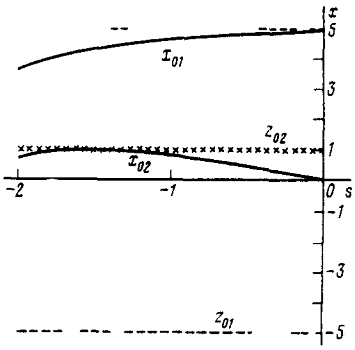

Fig.2

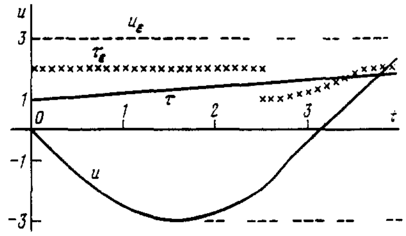

Fig.3

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THE METHOD OF PURSUIT BY SEVERAL CONTROLLED OBJECTS OF DIFFERENT TYPES*

N.L. GRIGORENKO

The problem of the pursuit of one evader by several controlled objects of different types is examined. The sufficient conditions are obtained for the pursuit game to terminate in a finite time. The proposed method of pursuer interaction assumes that the pursuing players are separated into two groups, the first of which holds the evader in some domain, while the second searches for the evader in this domain. The paper touches on the researches in /1—9/. Typical examples illustrate the results.

Let the motions of the vectors $z_1, \ldots, z_m$ in the n-dimensional Euclidean space R^n be described by the equations

$$z_i^{\cdot} = C_i z_i + u_i - v, \quad z_i(0) \approx z_i^{\circ}, \quad i = 1, \ldots, m \tag{1}$$

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